

Progression in Teaching Multiplication

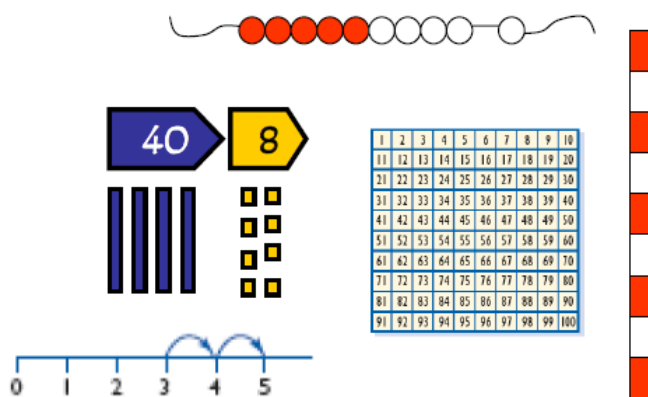
Mental Skills

- Recognise the size and position of numbers
- Count on in different steps 2s, 5s, 10s
- Double numbers up to 10
- Recognise multiplication as repeated addition
- Quick recall of multiplication facts
- Use known facts to derive associated facts
- Multiplying by 10, 100, 1000 and understanding the effect
- Multiplying by multiples of 10



Models and Images

- Counting apparatus
- Place value apparatus
- Arrays
- 100 squares
- Number tracks
- Numbered number lines
- Marked but unnumbered lines
- Empty number lines.
- Multiplication squares
- Counting stick
- Bead strings
- Models and Images charts
- ITPs-Multiplication grid, Number Dials, Multiplication Facts
- Numicon



In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

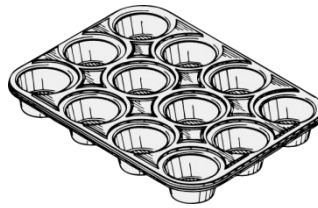
- the inverse of division
- commutative i.e. 5×3 is the same as 3×5
- associative i.e. $2 \times 3 \times 5$ is the same as $2 \times (3 \times 5)$

Early Learning Goal:

Children solve problems, including doubling.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays. **An array is objects or shapes arranged in a rectangle.**



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing the fingers on each hand as a double.



A child's jotting showing double three as three cookies on each plate.

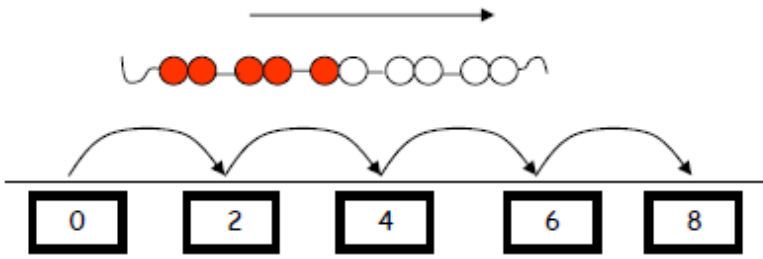
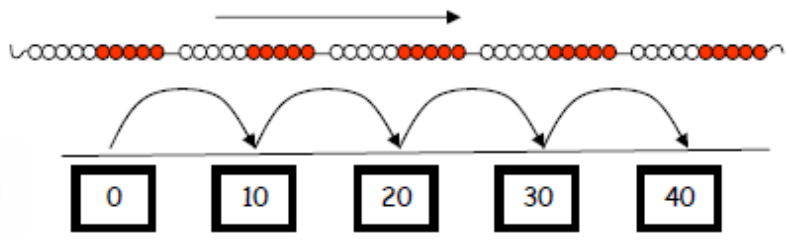
Y1

End of Year Objective:

Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In year one, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

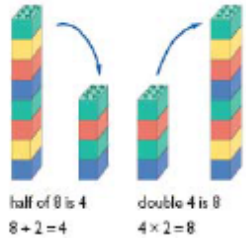
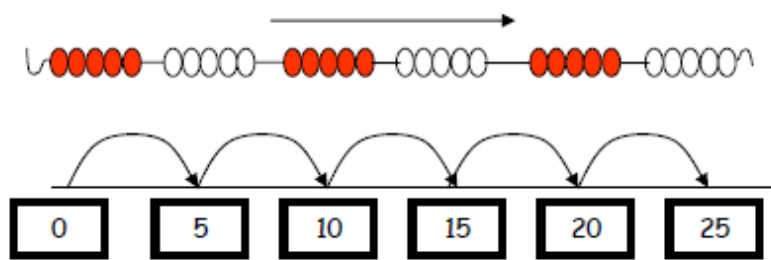
Count in tens from zero



Count in twos from zero



Count in fives from zero



Know doubles and corresponding halves

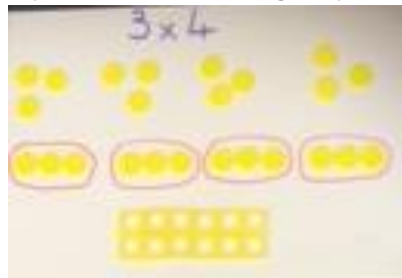
Y2

End of Year Objective:

Calculate mathematical statements for multiplication (using repeated addition) and write them using the multiplication (x) and equals (=) signs.

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g.

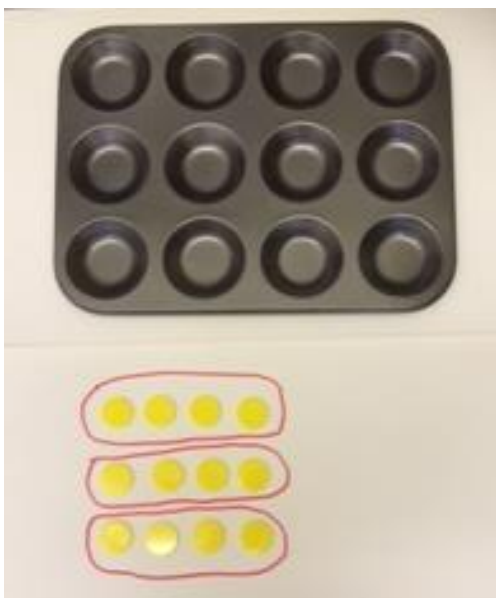
3×4 can be shown as four groups of three with counters, either grouped in a random pattern, or in a more ordered pattern, with the groups of three indicated by the border outline:



Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). **Again, children should be encouraged to use practical apparatus and jottings to support their understanding, e.g.**

$4 \times 3^*$ can be represented as an array in two forms (as it has commutativity)

An **array** is objects or shapes arranged in a rectangle. **Arrays are very important in helping children to understand how multiplication works. They learn that in the array the answer is always the same (eg 4×3 is the same as 3×4) even when you rotate.**



$$4 \times 3 = 4 + 4 + 4 = 12$$



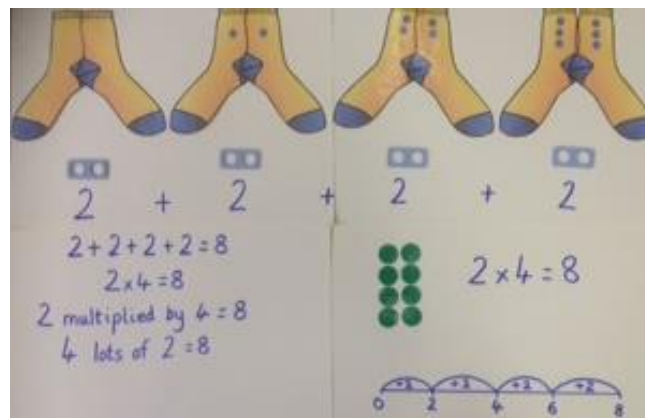
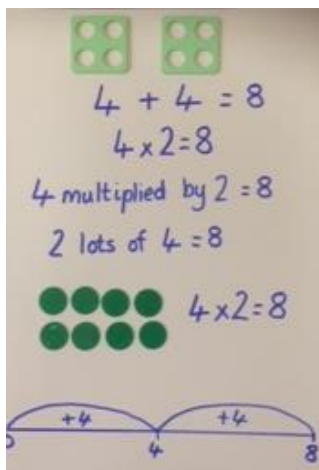
$$3 \times 4 = 3 + 3 + 3 + 3 = 12$$

*For mathematical accuracy 4×3 is four, three times and 3×4 is three, four times. Once children understand the commutative order of multiplication then the order is irrelevant.



Know by heart the facts for the 2, 5 and 10 times tables in any order.

Children should also understand multiplication as repeated addition. Children should also record pictorially as well as writing the number statement.



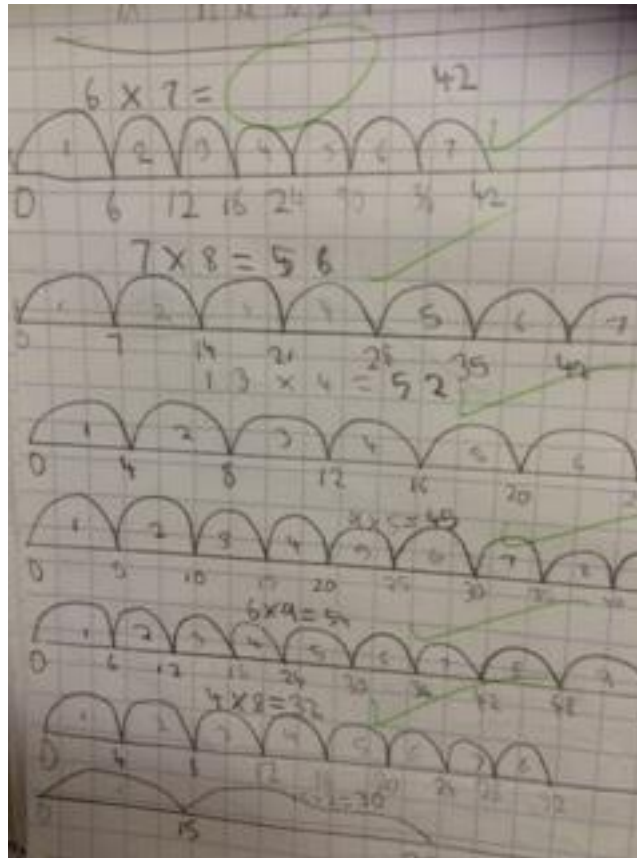
Y3

End of Year Objective:

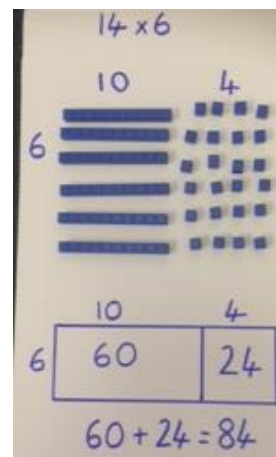
Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4 It is more beneficial for children's understanding to go through the practical methods of calculation as steps of development towards a formal written method.*

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g. 3×8 . Children can also use a numberline to work out multiplication tables.



As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and units/ones to help them. For example, when calculating 14×6 , children should set out the array, then partition the array so that one array has ten columns and the other four.



This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line. By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.

It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20

14 x 6 =
1. Partition 14 into tens and ones.

X	10	4	

14 x 6 =
1. Partition 14 into tens and ones
2. Write what you are multiplying by

X	10	4	
6			

14 x 6 =
1. Partition 14 into tens and ones.
2. Write what you are multiplying by.
3. Write both number statements

X	10	4	
6			
			14 x 6 =

14 x 6 =
1. Partition 14 into tens and ones.
2. Write what you are multiplying by.
3. Write both number statements.
4. Multiply the tens and write the answer

X	10	4	
6			
	60		14 x 6 =

14 x 6 =
1. Partition 14 into tens and ones.
2. Write what you are multiplying by.
3. Write both number statements.
4. Multiply the tens and write the answer.
4. Multiply the ones and write the answer

X	10	4	
6			
	60	24	14 x 6 =

14 x 6 =
1. Partition 14 into tens and ones.
2. Write what you are multiplying by.
3. Write both number statements.
4. Multiply the tens and write the answer.
4. Multiply the ones and write the answer.
5. Add the totals

X	10	4	
6			
	60	24	14 x 6 = 84

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Y4

End of Year Objective:
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Children will move to Y4 using whichever method they were using as they transitioned from Y3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

$$79 \times 8$$

x	70	9
8	560	72

$$\begin{array}{r} 560 \\ + 72 \\ \hline 632 \end{array}$$

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that $7 \times 8 = 56$ to know that $70 \times 8 = 560$.

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.

$$346 \times 8$$

x	300	40	6
8	2400	320	48

$$\begin{array}{r} 2400 \\ + 320 \\ + 48 \\ \hline 2768 \end{array}$$

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

The use of practical equipment should still be encouraged at this stage.

Y5

End of Year Objective:

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

$$4346 \times 8$$

x	4 000	300	40	6
8	32 000	2400	320	48

$$\begin{array}{r} 32000 \\ + 2400 \\ + 320 \\ + 48 \\ \hline 34768 \end{array}$$

and numbers with up to four digits by a two-digit number, e.g.

$$2693 \times 24$$

x	2000	600	90	3
20	40000	12000	1800	60
4	8000	2400	360	12

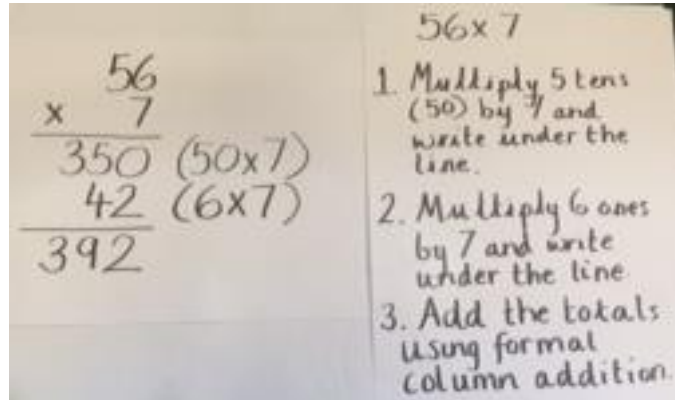
$$\begin{array}{r} 40000 \\ + 8000 \\ + 12000 \\ + 2400 \\ + 1800 \\ + 360 \\ + 60 \\ + 12 \\ \hline 64632 \end{array}$$

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Linking the grid method and formal algorithm

x	50	6	
7	350	42	



When children are working with decimals, they can continue to link the grid to the above method. Books cost £1.49. How much will it cost for 7 books.

x	1	0.4	0.09	
7	7	2.80	0.63	$7 + 2.80 + 0.63$

1.49

x 7

7.00

2.80

0.63

1

Answer $1.49 \times 7 =$

10.43

Y6

End of Year Objective:

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.

Multiply one-digit numbers with up to two decimal places by whole numbers.

By the end of Y6, children should be able to use the standard method to multiply any number by a two-digit number or a decimal.

Continue to link the grid method to the formal algorithm until children are ready to use only the standard written method.

x	200	40	6
30	6000	1200	180
7	1400	280	42

$6000+1200+180 = 7380$

→

7380

$1400+280+42 = 1722$

→

$\underline{1722}$

Answer $246 \times 37 = 9102$

246
 $\times \underline{37}$
 7380
 $\underline{1722}$
 9102

Videos to support the transition from the grid to the formal method can be found on the NCETM website. (*National Centre of Excellence in Teaching Mathematics*).

<http://vimeo.com/70318365>